Naval Research Laboratory

Washington, DC 20375-5000

NRL Memorandum Report 6579

Scaling Z-Pinch Plasmas With Atomic Number for a Given K-Shell Emission

K.G. WHITNEY, J.P. APRUZESE, J.W. THORNHILL AND J. DAVIS

Radiation Hydrodynamics Branch
Plasma Physics Division

November 17, 1989

This research was sponsored by the Defense Nuclear Agency under Project Task Code and Title, RL RA/Advanced Technology Development, Work Unit Code and Title, 00079, Advanced Technology Development, MIPR No. 89-565.

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SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

Form Approved OMB No. 0704-0188

1a REPORT SECURITY CLASSIFICATION: UNCLASSIFIED	1b RESTRICTIVE MARKINGS
2a SECURITY CLASSIFICATION AUTHORITY	3 DISTRIBUTION/AVAILABILITY OF REPORT: Approved for public release; distribution unlimited.
2b DECLASSIFICATION/DOWNGRADING SCHEDULE	
4 PERFORMING ORGANIZATION REPORT NUMBER(S): NRL Memorandum Report 6579	5 MONITORING ORGANIZATION REPORT NUMBER(S)
6a NAME OF PERFORMING ORGANIZATION: Naval Research Laboratory	6b OFFICE SYMBOL (If applicable): Code 4720
6c ADDRESS (City, State, and ZIP Code): Washington, DC 20375-5000	7b ADDRESS (City, State, and ZIP Code)
8a NAME OF FUNDING/SPONSORING ORGANIZATION: Defense Nuclear Agency	8b OFFICE SYMBOL (If applicable): RAEV
8c ADDRESS (City, State, and ZIP Code): Alexandria, VA 22310	10 SOURCE OF FUNDING NUMBERS: PROGRAM ELEMENT NO: 62715N; PROJECT NO; TASK NO; WORK UNIT ACCESSION NO: DN880-191

11 TITLE (Include Security Classification): Scaling Z-Pinch Plasmas With Atomic Number for a Given K-Shell Emission

12 PERSONAL AUTHOR(S): Whitney, K.G., Apruzese, J.P., Thornhill, J.W. and Davis, J.

13a TYPE OF REPORT	13b TIME COVERED	14 DATE OF REPORT (Year Month Day)	15 PAGE COUNT
Interim	FROM _____ TO _____	1989 November 17	35

16 SUPPLEMENTARY NOTATION: This research was sponsored by the Defense Nuclear Agency under Project Task Code and Title, RL RA/Advanced Technology Development, Work Unit Code and Title, 00079, Advanced Technology Development, MIPR No. 89-503.

17 COSATI CODES			18 SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	
			Z-pinch; Plasmas; Emission; X-radiation

19 ABSTRACT (Continue on reverse if necessary and identify by block number)

Simple scaling arguments are developed to show how specification of a desired total K-shell emission can be used to determine the parameters of an imploding array z-pinch plasma as a function of the atomic number of the pinch. The model we use ignores all hydrodynamic details of the z-pinch implosion and thermalization processes and assumes that all of the plasma mass is imploded and thermalized. The pinch is also assumed to be driven by a prescribed linearly rising current. The scaling laws that are derived from these assumptions, in effect, contain phenomenological scaling parameters whose values will need to be more accurately determined by more detailed theoretical calculations and/or by z-pinch experiments on a variety of different machines.

20 DISTRIBUTION/AVAILABILITY OF ABSTRACT: ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT ☐ DTIC USERS	21 ABSTRACT SECURITY CLASSIFICATION: UNCLASSIFIED
22a NAME OF RESPONSIBLE INDIVIDUAL: Dr. Kenneth Whitney	22b TELEPHONE (Include Area Code): (202) 767-2921; 22c OFFICE SYMBOL: Code 4720

DD Form 1473, JUN 86 — Previous editions are obsolete — SECURITY CLASSIFICATION OF THIS PAGE

S/N 0102-LF-014-6603

CONTENTS

SCALING Z-PINCH PLASMAS WITH ATOMIC NUMBER FOR A GIVEN K-SHELL EMISSION

I. INTRODUCTION

A major goal in utilizing z-pinch plasmas as high brightness laboratory sources of x-rays for such diverse applications as x-ray lithography, x-ray lasers, materials studies, and plasma spectroscopy is to increase the radiation emission above about 1 keV through the use of moderate atomic number (Z = 10 to 36) materials. Two kinds of configuration, single wires and wire arrays or foils[1-8], have been investigated in the past. Gas puffs can also be used in either geometry though only crudely since the supersonic gas flow at best approximates these shapes. In both cases, it has been observed that optimal x-ray emission decreases on a given machine as the atomic number of the z-pinch material is increased[2,9].

Single wire dynamics consists of a complicated interplay between electrical, MHD, ionization, and radiation phenomena. The question, therefore, of how the x-ray emission of these materials will scale with atomic number is, in principle, a difficult one. Wire arrays, on the other hand, initially have a comparatively simple dynamics (to zeroth approximation). A large fraction of the electrical energy is first converted into the kinetic energy of implosion, which is then thermalized when the array collides with itself at the axis of the pinch. In this case, a force and a circuit equation suffice to describe the optimal electrical coupling of the z-pinch load to the pulse power machine[10]. In this paper, we will describe how the model of Ref. 10 can be modified and extended to provide predictions of how machine and diode designs must be scaled in order to maintain a given K-shell x-ray emission as the plasma atomic number increases in z-pinch array implosions.

II. THE MODEL

Let m denote the mass per unit length of the wire array or foil and let r(t) be the outside radius of the circular array as a function of time. In response to the magnetic forces that are generated when a current I(t) flows through the array, it accelerates inward:

Manuscript approved April 26, 1989.

$$m \frac{d^2 r}{dt^2} = \frac{1}{2} \frac{\partial L'}{\partial r} I^2 \qquad (1)$$

where L' is the array load inductance per unit length[10]. For an array consisting of N wires

$$\frac{\partial L'}{\partial r} = - \frac{N-1}{N} \frac{\mu_o}{2\pi} \frac{1}{r} \quad . \qquad (2)$$

Following Ref. 10, we will utilize the dimensionless variables,

$$x = r/r_o, \qquad \tau = t/t_o, \qquad i = I/I_o, \qquad (3)$$

where r_o is the initial array radius and

$$t_o = \frac{N-1}{N} \frac{\mu_o \ell}{2\pi R_o} \simeq \frac{L_o}{R_o} \quad . \qquad (4)$$

I_o is the maximum current that flows in the load, ℓ is the length of the array load, R_o is the diode resistance, and $L_o \equiv \mu_o \ell/(2\pi)$. In terms of these variables, Eq. (1) becomes

$$\frac{d^2 x}{d\tau^2} = - A \frac{i^2}{x} \quad . \qquad (5)$$

where, for large N, the dimensionless constant A is given by

$$A \simeq \frac{\frac{1}{2}(L_o/\ell) I_o^2}{m(r_o/t_o)^2} \qquad (6)$$

For a wide variety of existing pulse power machines, the current pulse is adequately described by a linear ramp $I(t) = \dot{I}_o t$ during the main run-in phase of the array implosion. In pulse power machines that are designed as current sources, this should also be a useful description of the current pulse. From Eq. (6), one can derive an equation for the increase of kinetic energy with implosion distance:

$$\frac{1}{2}\left(\frac{dx}{d\tau}\right)^2 = A\int_x^1 \frac{i^2}{x'}\,dx' \quad . \tag{7}$$

It is clear from Eq. (7) that maximum kinetic energy is generated in the implosion if the current is large in the final stages of the run-in. Thus, on both experimental and theoretical grounds, it is useful to investigate the scaling of z pinch implosions for linearly rising current pulses. For this case, one can define a new dimensionless quantity q by

$$q = q_o\,\tau \ , \tag{8}$$

where

$$q_o \equiv A^{1/4}\,(\dot{I}_o t_o/I_o)^{1/2} \ , \tag{9}$$

so that Eq. (5) simplifies to[11]

$$x\,\frac{d^2x}{dq^2} = -\,q^2. \tag{10}$$

The picture of array implosions that is provided by Eq. (10) eventually breaks down when the plasma begins to collide on itself and to thermalize the kinetic energy generated prior to this collision. Experimental evidence[10] on implosion times obtained from a number of machines suggests, in fact, that Eq. (10) is generally valid until the plasma has collapsed to about 10% of its initial radius, at which time further collapse is no longer discernible and the main radiation pulse commences. Thus, we will be interested in the solution to Eq. (10) up to values of $x \sim 0.1$ subject to the boundary conditions that the implosion begins at rest $dx/d\tau = 0$ when $x = 1$.

Eq. (10) can be solved either analytically by power series or numerically. The solution information of interest is the final values of q and (dx/dq), q_f and $(dx/dq)_f$ respectively, that are achieved when x reaches its final value x_f just prior to the thermalization process. These quantities, obtained from numerical solutions to Eq. (10), are shown in Figures 1 and 2. Least squares fits to the computed values of q_f and $(dx/dq)_f$ are also drawn in these figures. These fits are given by the

polynomial expressions,

$$q_f = 1.73 - 0.22\, x_f - 0.42\, x_f^2 \quad , \tag{11}$$

$$(dx/dq)_f = 4.72 - 18.9\, x_f + 49.1\, x_f^2 \quad , \tag{12}$$

valid for $0.04 \leq x_f \leq 0.14$.

III. Scaling Arguments

By assuming a linear rise in current, one guarantees that the time of array implosion t_f will coincide with the time of peak current. What we would like to require, in addition, is that during the implosion, the array acquires enough kinetic energy to ignite the K-shell and contains enough mass to produce the K-shell emission rates that are required of a high yield super-kilovolt x-ray source. In the process, we would like to determine how these requirements scale with atomic number, Z, from Z = 10 (neon) to Z = 36 (krypton).

To begin, consider a uniform plasma cylinder of unit length and radius r_f containing a single element of atomic number Z. The amount of mass m contained within this cylinder is

$$m = A m_p N_i \pi r_f^2, \tag{13}$$

where A is the atomic weight of the element, m_p is the mass of a proton, and N_i is the ion density of the plasma. A least squares polynomial fit to log A as a function of Z results in

$$A \tilde{=} 1.58\, Z^{1.1}. \tag{14}$$

This functional fit, up to Z = 36, to the atomic weights is shown in Figure 3. Thus

$$m(\mu g/cm) = 8.28 \times 10^{-18} N_i r_f^2 Z^{1.1} \tag{15}$$

In this paper, we will employ a mixed set of MKS units that are appropriate to a discussion of z-pinches. Thus, we will generally express distances and ion densities in units of centimeters and cm^{-3}, masses in units of

micrograms, yields in kilojoules, energies and temperatures in eV, currents in mega-amperes, and time in nanoseconds or microseconds.

There are two ways to determine the scaling of K-shell yield with atomic number. One is by a simple theoretical argument, the other is by an analysis of previous and future experimental and theoretical data. Because of its large oscillator strength, the He-α $1s^2$-1s2p line dominates the K-shell emission from low density, optically thin plasmas. Thus, one can argue on the basis of the optically thin emission rate of this line that the yield per unit length, y, of total K-shell emission will scale as a power of N_i and inversely as a power of Z. The coefficient of proportionality C_y can be determined either from direct spectroscopic measurements of N_i and T_e made in previous low atomic number z-pinch shots for which an optimal K-shell yield was approximately obtained or from theoretical calculations. One then finds that

$$y(kJ/cm) \tilde{=} C_y N_i^{\alpha} Z^{-\beta} E(Z) r_f^{\gamma} \qquad (16)$$

where E(Z) is an exponential function of Z to be defined shortly.

Estimates for the exponents, α, β, and γ can be obtained from the following coronal equilibrium arguments. As the atomic number of a given sized plasma is increased, the photon probabilities of escape in the resonance lines of helium or hydrogen approach 1. Thus, the K-shell radiation rates of the plasma become increasingly optically thin. In this case, the amount of He-α emission from our uniform plasma cylinder of unit length will be

$$y_{He-\alpha} = N_{He} N_e X_{1s-2p} \Delta E \pi r_f^2 \tau_K \qquad (17)$$

where N_{He} is the number of ions in the helium-like ground state, N_e is the electron density, X_{1s-2p} is the rate coefficient for exciting the $1s2p\,^1P$ state from the ground state, ΔE is the energy of the transition and of the radiated photon, and τ_K is the duration of the emission. To determine the scaling of $y_{He-\alpha}$ with N_i and Z we use Seaton's formula for the collisional excitation rate for dipole allowed transitions[12]:

$$X_{1s-2p}(cm^3/sec) = 1.6\times10^{-5}\ fg\ \frac{e^{-\Delta E/T_e}}{\Delta E\ T_e^{1/2}}\ , \qquad (18)$$

where f is the oscillator strength, g is the thermally averaged gaunt factor ~ 1, and T_e is in units of eV. The temperature T_e for which K-shell occupation and excitation is strong does not scale, as ΔE does, with Z^2 because of the different Z dependences of the collisional ionization and radiation recombination rates. Based on estimates of ionization equilibrium in hydrogen, we will take this scaling to be

$$T_e(eV) \tilde{=} 0.3\ Z^{2.9}, \qquad (19)$$

although a more detailed analysis of this scaling should be made. Thus,

$$X_{1s-2p} \sim E(Z)\ /\ Z^{3.45}, \qquad (20)$$

where, because $\Delta E \sim Z^2$ and $T_e \sim Z^{2.9}$, we take $E(Z) = \exp(-20.6/Z^{0.9})$. In order to determine the scaling of τ_K with Z, we assume that K-shell emission is proportional to the inertial confinement time of the plasma during emission:

$$\tau_K \sim \frac{r_f}{v_{th}} = \frac{r_f}{(2kT_e/Am_p)^{1/2}} \sim r_f/\ Z^{0.9}. \qquad (21)$$

Thus, in coronal approximation,

$$y_{He-\alpha} \sim \frac{N_i^2 r_f^3\ E(Z)}{Z^{1.35}}\ , \qquad (22)$$

and, in this case, taking y and $y_{He-\alpha}$ to scale identically with Z and N_i we find that $\alpha \tilde{=} 2$, $\beta \tilde{=} 1.35$, and $\gamma \tilde{=} 3$.

If the K-shell of a given element is to be excited, a minimum energy must be imparted to each imploding ion in order to (1) strip all but one or two electrons from the ion and (2) maintain each stripped electron at

temperatures to avoid recombination back into the L-shell. Of course inefficiencies in the thermalization process, in particular subkilovolt radiative losses, imply that much more energy than this is needed in practice. However, an initial insight into basic Z-scaling may be obtained by consideration of these basic quantities.

As noted, the temperatures at which maximum amounts of K-shell radiation are expected are $T_e \approx 0.3\ Z^{2.9}$. We approximate the number of stripped electrons as equalling Z. Therefore, the thermal energy of the stripped electrons from an ion of atomic number Z at this temperature is,

$$E_t = 0.4\ Z^{3.9}\ \text{eV/ion}. \tag{23}$$

The energy needed to strip the electrons from each ion is also important and non-negligible. We shall assume that the plasma is 50% hydrogen-like and 50% helium-like. The ionization potentials for each element are available from many references; we have used the data of Carlson et al.[13] The energy needed is given by the following fit - which for purposes of the approximate treatment of this paper has negligible error:

$$E_s = 3.83\ Z^{2.64}\ \text{eV/ion}. \tag{24}$$

Obviously, the minimum energy per ion that must be acquired during thermalization is given by E_t+E_s. A critical question is, in practice however, how much energy per ion beyond the minimum is required for optimum K-shell radiation to be generated in a z-pinch? We will assume that the kinetic energy per ion that must be generated in an array implosion is some multiple, ε, of this minimum energy. From past analyses of aluminum experimental data, there is some indication that $\varepsilon \sim 15$ and that it may be as large as 30. We now make the assumption that, for all Z, ε (~ 15) times (E_t+E_s) will be required for efficient K-shell radiation production. Such a large factor is plausible when one considers the numerous sinks for generator energy other than production of K-shell radiation. Such processes include runaway electron production during the implosion, the generation of axial plasma motions, nonuniform thermalization and radiative losses other than K-shell. This is not unreasonable as a first approximation since it allows radiative and other losses to scale upward with Z in the same fashion as the thermal and ionization energies, which is

initially reasonable. Beginning with a factor of 15 can be viewed optimistically, since it indicates that improvements in the efficiency of these devices may be achieveable. This assumption means that

$$\frac{1}{2} m_i (dr/dt)_f^2 = (KE)_{ion} = 1.6 \times 10^{-19} \varepsilon (0.4\, Z^{3.9} + 3.83\, Z^{2.64})$$

$$= 2.39 \times 10^{-19}\, \varepsilon\, Z^{3.51}\ (J/ion), \qquad (25)$$

where, in Eq. (25), the additional simplification of the power-law sum to a single power law does not result in substantial error for present purposes.

IV. RESULTS

Equations (11), (12), (14), (15), (16), (19), and (25) allow one to determine the scaling of the mass loading, implosion time, and the rate of current rise, $\dot{I}_o$ (I-dot), with atomic number in order to achieve a given K-shell yield. For example, on eliminating N_i from Eqs. (15) and (16), one finds that

$$m\ (\mu g/cm) = C_m y^{1/\alpha} r_f^{(2-3/\alpha)} Z^{(1.1+\beta/\alpha)} E(Z)^{-1/\alpha}, \qquad (26)$$

where $C_m = 8.28 \times 10^{-18}/C_y^{1/\alpha}$. The implosion time is $t_f = (t_o/q_o)\, q_f$; it can be found from Eqs (11), (12), and (25) since

$$(dr/dt)_f = \frac{q_o r_o}{t_o} (dx/dq)_f = \frac{q_f r_o}{t_f} (dx/dq)_f. \qquad (27)$$

Thus,

$$t_f = q_f \left(\frac{dx}{dq} \right)_f \left(\frac{m_i}{2(KE)_{ion}} \right) r_o\ , \qquad (28)$$

where we now write $(dx/dq)_f$ for the magnitude of this final velocity. Finally, we note that the I-dot of the implosion can be determined from Eqs. (6), (9), (28) and the relationship $t_f/q_f = t_o/q_o$:

$$\frac{t_o}{q_o} = \left(\frac{m r_o^2}{1/2\ L_o \overset{\circ}{I}_o^2} \right)^{1/4} = \frac{t_f}{q_f} = \left(\frac{dx}{dq}\right)_f \left(\frac{m_i}{2(KE)_{ion}}\right)^{1/2} r_o, \qquad (29)$$

i.e,

$$\overset{\circ}{I}_o = \left(\frac{4\pi m}{\mu_o}\right)^{1/2} \left\{\frac{q_f^2 r_o}{t_f^2}\right\} = \left\{\frac{2(KE)_{ion}}{m_i r_o}\right\} \left(\frac{4\pi m}{\mu_o}\right)^{1/2} \left(\frac{dx}{dq}\right)_f^2 . \qquad (30)$$

For a given yield and initial array radius, all of the quantities on the right hand sides of Eqs. (26), (28), and (30) are scalable with Z, assuming that conventional wisdom is correct and back EMF's and plasma pressure forces will stop the 1-d implosion and begin to thermalize the kinetic energy at or around a value of $x_f \sim 0.1$ (see Ref. 10). In order to illustrate this procedure in more detail, we will now specialize to two cases where α=2, β=1.35, and γ=3 (see Eq. 22) and, from experimental estimates, we take C_y to be 9.1×10^{-33}. Eqs. (26), (28), and (30) then become

$$m\ (\mu g/cm) = 8.68 \times 10^{-2} (yr)_f^{1/2} Z^{1.776} \exp(9.2/Z^{0.9}), \qquad (31)$$

$$t_f(ns) = 7.5 \times 10^2\ q_f\ (dx/dq)_f\ r_o Z^{-1.205}\ \varepsilon^{-0.5}, \qquad (32)$$

$$\overset{\circ}{I}_o(MA/\mu s) = 4.44 \times 10^{-5}\ (dx/dq)_f^2\ \varepsilon\ \frac{(yr_f)^{1/4}\ Z^{3.3}}{r_o}\ \exp(4.6/Z^{0.9}), \qquad (33)$$

where as mentioned earlier, y is expressed in units of kJ/cm and r_o and r_f in units of cm. Graphs of these scaling relations are shown in Figures 4-6 for two cases.

In case I, we took the initial radius of the array to be 1 cm, and assumed x_f=0.1, so that q_f=1.7, $(dx/dq)_f$=3.3, and r_f=0.1 cm. The desired yield was taken to be 2kJ/cm and the thermalization efficiency, ε=15. In

case II, these values were taken to be: r_o=2 cm, x_f=0.07, q_f=1.71, $(dx/dq)_f$=3.63, r_f=0.14 cm, y=15 kJ/cm, and ε=10.

It is clear from Eqs. (31) - (33) and from Figs. 4-6 that array implosions must start at a large radius and implode to a fairly tight cylinder on axis with a good kinetic-to-thermal conversion efficiency in order to produce high K-shell yields with long implosion times and small I-dots. Moreover, for a given machine, with given I-dot and current capabilities, the optimum choice of element depends on the yield-per-length and spectrum requirements. It is also clear, however, that the pulse power machine must be redesigned as the load element or yield requirement is changed.

The above scaling relations derive from the assumptions of an optically thin plasma and an impressed linearly rising current across the plasma that terminates at the time of kinetic energy thermalization. Figure 7, which shows the ion densities that are computed from Eq. (15) and (31), indicates that the first assumption holds up in the two cases under study very well. As to the second assumption, we replaced the voltage source utilized in Katzenstein's work[10] with a current source with obvious fundamental consequences, since array implosions are current driven.

In general, the ohmic resistance of the array plasma is small, so that the voltage drop across the load $V_\ell(t)$ is given by $V_\ell = d(L_\ell I)/dt$, where the load inductance L_ℓ for an N-wire array of wire diameter d_w having a return current path radius R is given by[10]:

$$L_\ell = L_o \left(\frac{1}{N} \ln \left(\frac{R}{N d_w} \right) + \frac{N-1}{N} \ln \left(\frac{R}{r} \right) \right) \tag{34}$$

Hence

$$\frac{dL_\ell}{dt} \simeq L_o \frac{1}{r} \left(- \frac{dr}{dt} \right) , \tag{35}$$

and

$$V_\ell = \dot{I}_o \left(L_\ell + L_o \, (t/r)(-dr/dt) \right). \tag{36}$$

After converting to dimensionless variables, one finds that the voltage drop across the load at the end of the implosion, $V_{\ell f}$, is

$$V_{\ell f} = \left(L_\ell + (q_f/x_f)(dx/dq)_f\, L_o\right) \dot{I}_o \quad . \tag{37}$$

By virtue of Eq. (33), therefore, it follows that $V_{\ell f}$ scales as $Z^{3.3}$.

In general, the second term in Eq. (37) dominates over the first. As the load collapses, therefore, maintenance of a steadily rising current requires a disproportionately large rise in the voltage across the plasma near the end of the collapse. This point is demonstrated in Figures 8 and 9. We have defined the peak current I_o to be $\dot{I}_o t_f$. For an array length of 2 cm, we see from Figures 8 and 9 that peak currents of near 2 MA in aluminum generate peak voltages at the end of the collapse near 7 MV. For selenium, however, peak currents of 11 MA and 14 MA are needed in the two cases and they produce peak voltages near 150 MV. Under these conditions, pulse power machines will not act as current sources and the current will generally take a steep dip at the time of the final collapse of the plasma on axis, rendering the assumption of a linear current rise invalid. For this reason, the scaling relations developed in this paper will generally need some correcting and/or to be empirically determined under experimental conditions that implode the arrays in the required times and give them the required final kinetic energy per ion.

One theoretical feature of the Eqs. (19) - (33) scaling relations, already has some experimental support: that of the scaling of yield with peak current for optimal yields. Since $I_o = \dot{I}_o t_f$, it follow from Eq (30) that

$$m(\mathrm{kg/m}) = \frac{\mu_o}{4\pi}\left\{\frac{t_f}{q_f^2 r_o}\right\}^2 I_o^2. \tag{38}$$

Therefore, from Eqs. (13) and (16) (with α=2, β=1.35, and γ=3) one finds that

$$y(\mathrm{kJ/cm}) = 2.6\times10^{-16}\, E(Z)\left\{\frac{I_o^4}{r_f Z^{8.37}}\right\} \quad . \tag{39}$$

Thus, the scaling relations can be rewritten to exhibit an I^4 scaling of the yield on peak current. In the context of a fixed pulse power machine, such scaling has been observed experimentally for optimal K-shell yields in neon.[14]

It is also of interest to determine what the scalings of Eq. (31) - (33) predict on average about the balance of pressure, f_p, and magnetic field, f_{jxB}, forces per unit length at the end of the implosion. Using Eqs. (13) and (19), we find (in MKS units) that

$$f_p \equiv 2\pi r_f p_f \tilde{=} 2\pi r_f Z N_i k T_e$$

$$= 4.8\times10^{-20}\ (2m/(m_i r_f))Z^{3.9}. \qquad (40)$$

Also, from Eqs. (25), (28), and (30),

$$f_{jxB} \equiv \frac{\mu_o I_o^2}{4\pi r_f} =$$

$$= 2.39\times10^{-19}\left(\frac{2m}{m_i r_f}\right)\frac{q_f^2}{(dx/dq)_f^2}\ \varepsilon\ Z^{3.51}\ . \qquad (41)$$

These forces will be equal only for a particular thermalization efficiency

$$\varepsilon = 0.2\left(dx/dq\right)_f^2 Z^{0.39}/\ q_f^2, \qquad (42)$$

which, in the two cases under consideration in this paper, has the values (<4) displayed in Figure 10. Since ε is expected to have values much larger than those in Figure 10, f_{jxB} will be much larger than f_p. However, as noted above, in reality, the current will dip sharply at the moment the plasma arrives on axis. Thus, the collapse will be stopped as the forces reverse in strength. This accounts, to some extent, for the empirical observation that $x_f \sim 0.1$. Plasma instability development may also contribute to this effect.

V. Discussion and Conclusions

The basic idea underlying the scaling relations of Eqs. (31) - (33) is that if z-pinch x-ray sources with a given yield and successively higher kilovolt energies are to be developed, then one must redesign the pulse power machine as the atomic number of the plasma is increased in order to implode the required higher plasma mass with a higher power input and a shorter implosion time (for a given plasma load geometry). It follows, conversely, that if the machine is not redesigned or reconfigured to the plasma, that one can expect the yield of K-shell x-rays to fall as the atomic number of the plasma is increased in conformity with experimental observations.[9] It also follows that the theoretical convenience of a linearly rising current pulse should not be strictly required experimentally, provided the required mass can be imploded to the required final kinetic energy by an appropriate circuit design.

The more fundamental scaling relations of this paper are given by Eqs. (26), (28), and (30) with α, β, γ, ε, x_f, q_f, and $(dx/dq)_f$ regarded as empirical constants to be determined ultimately, either from experimental data or from 1-d hydrodynamic computer simulations of the array or foil implosion process. Simulations are needed, in particular, in order to determine (1) the thermalization efficiency, ε, of foil implosions and (2) whether or not this efficiency itself is a scalable function of Z and/or a function of yield. We carried out some preliminary 1-d calculations for a titanium foil. They suggest that thermalization efficiencies of 15 do occur for a certain class of implosions. One major limitation of these calculations is that they do not describe the instability growth that is known to occur experimentally, and that undoubtedly plays a role in limiting the application of the above theory and the extent of the implosion (i.e. the size of x_f) and, thus, the thermalization efficiency. However, it should also be noted that experiments conducted to date, in which instability growth was observed and in which the implosion was limited, were generally non-optimally designed (as described in this paper) for radiation production in the K-shell. Another factor that may limit the range of validity of these scaling relations is the tendency of z-pinches to undergo radiative collapse.[15,16] If the mass loading is sufficiently

high or if instabilities form, the plasma will implode to radii much smaller than 0.1 r_o.

Finally, important experimental evidence of non-optimum performance is frequently provided by filtered pinhole pictures of the z-pinch emission region.[2,3] It is generally observed that as the energy of the photons increases, the size of the corresponding emission region decreases. Depending on the pulse power machine, this generally means that as Z increases, the fraction of the foil mass that participates in the K-shell emission decreases. In this sense, Z-pinch plasmas, like vacuum sparks, are remarkably accommodating. When more plasma than can be ionized to the K-shell is imploded, only an appropriate fraction, matched to the power capabilities of the circuit, is heated to produce the K-shell emission. This experimental behavior contrasts with one of the basic assumptions of the model calculations of this paper that all of the foil mass that is accelerated inward participates in the subsequent thermalization and K-shell emission. Any experiments that are conducted to test the scaling relations proposed in this paper should carefully examine the validity of this mass participation assumption.[17]

VI. Acknowledgements

This work was supported by the Defense Nuclear Agency and the Office of Naval Research.

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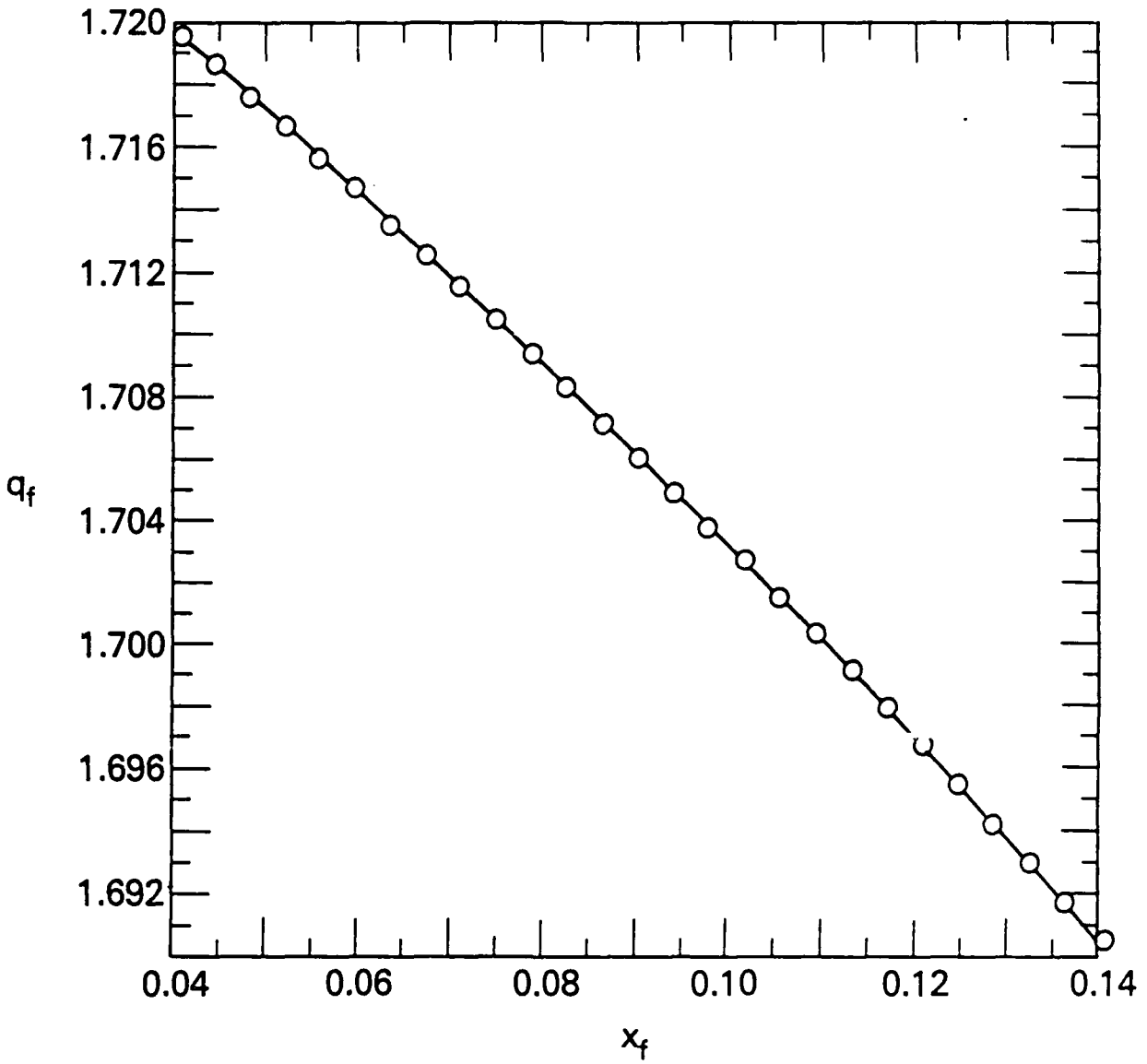

Figure 1. A least squares polynomial fit to calculated values of q_f is shown valid for values of x_f between 0.04 and 0.14.

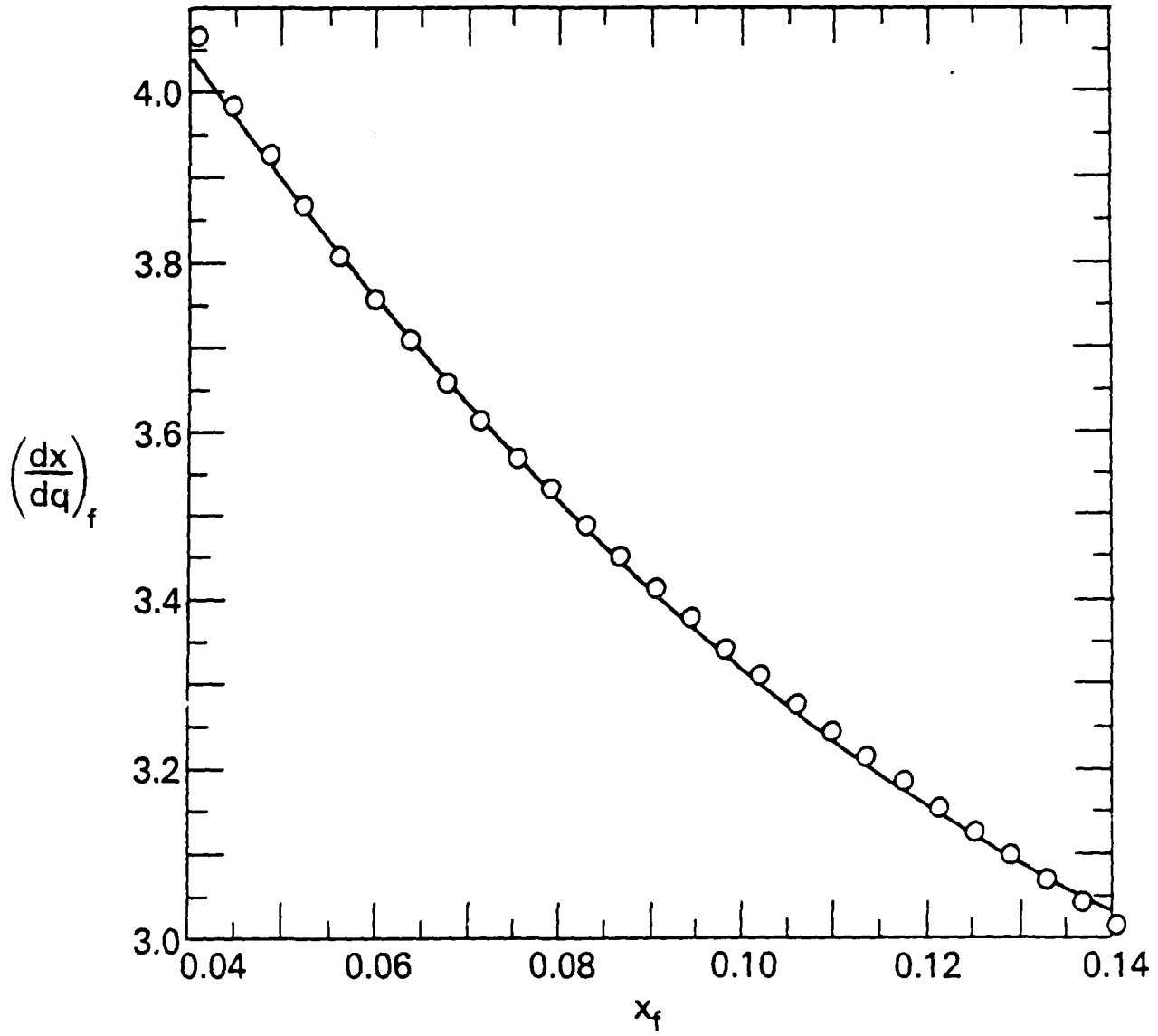

Figure 2. A least squares polynomial fit to calculated values of $(dx/dq)_f$ is shown valid for values of x_f between 0.04 and 0.14.

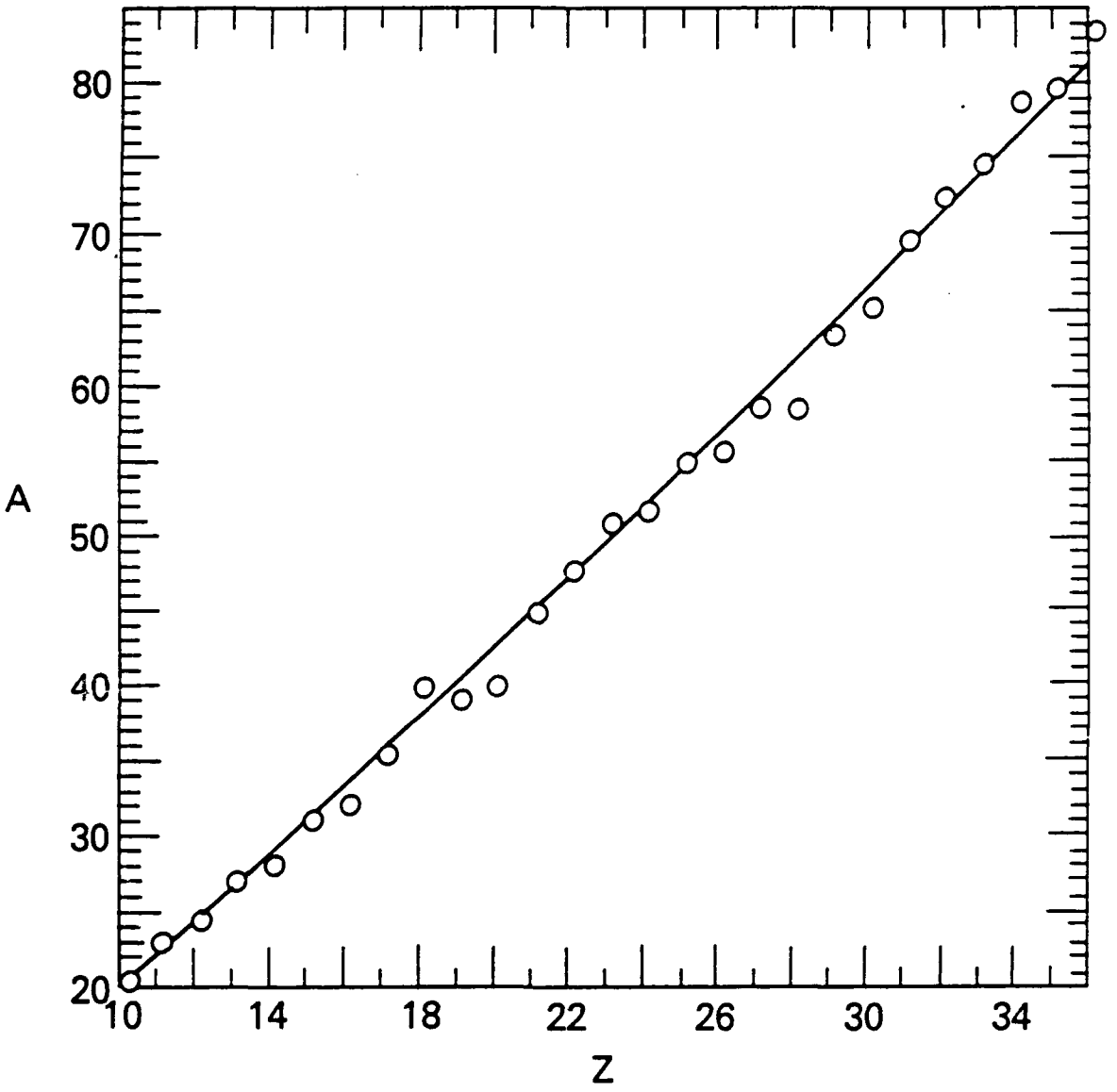

Figure 3. The atomic weights between neon (Z=10) and krypton (Z=36) are drawn relative to the curve fit $A=1.58Z^{1.1}$.

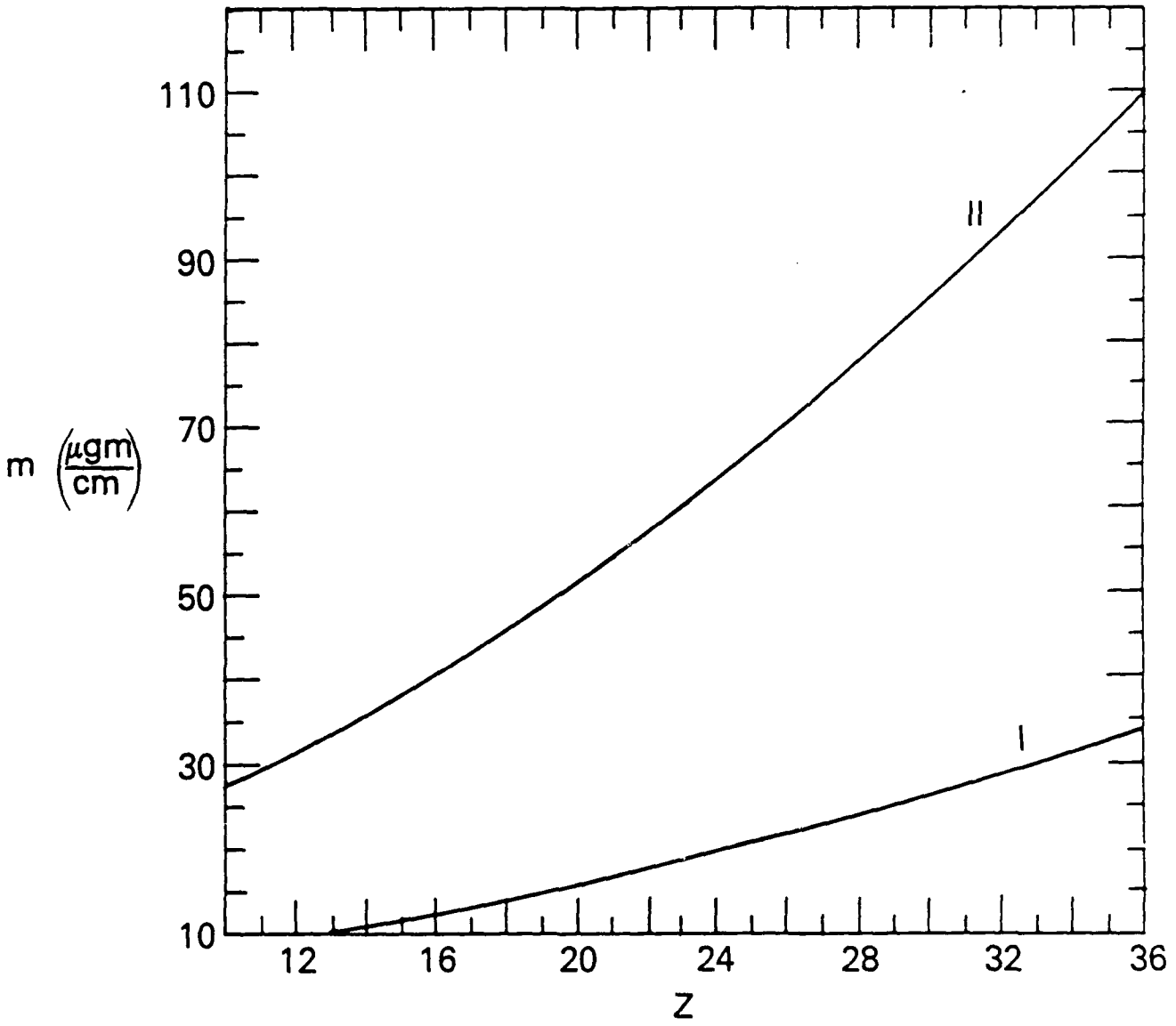

Figure 4. The scaling of the mass-per-length of a z-pinch array as a function of atomic number is shown for the two cases (I and II) described in the text.

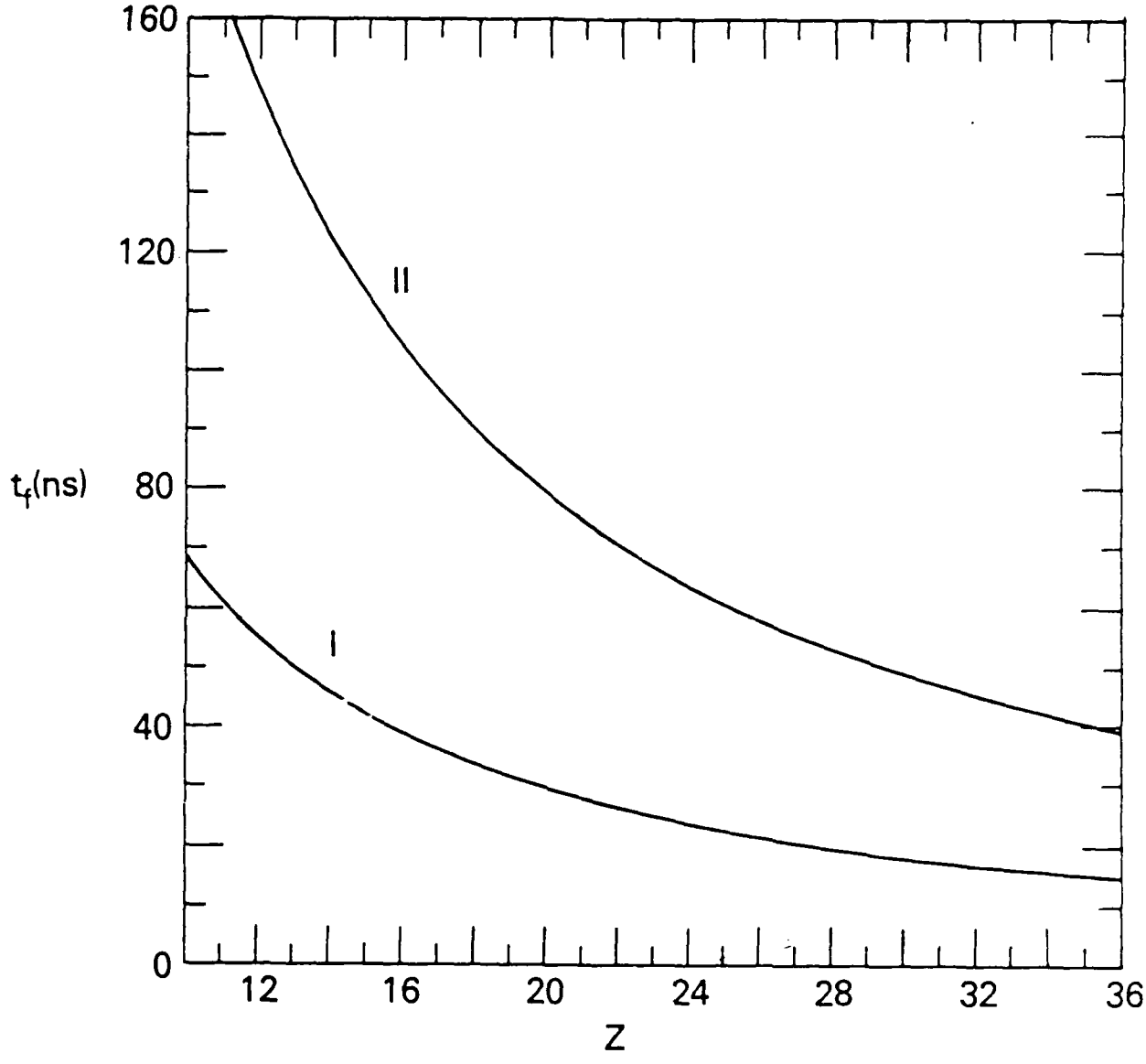

Figure 5. The scaling of the array implosion time as a function of atomic number is shown for the two cases (I and II) described in the text.

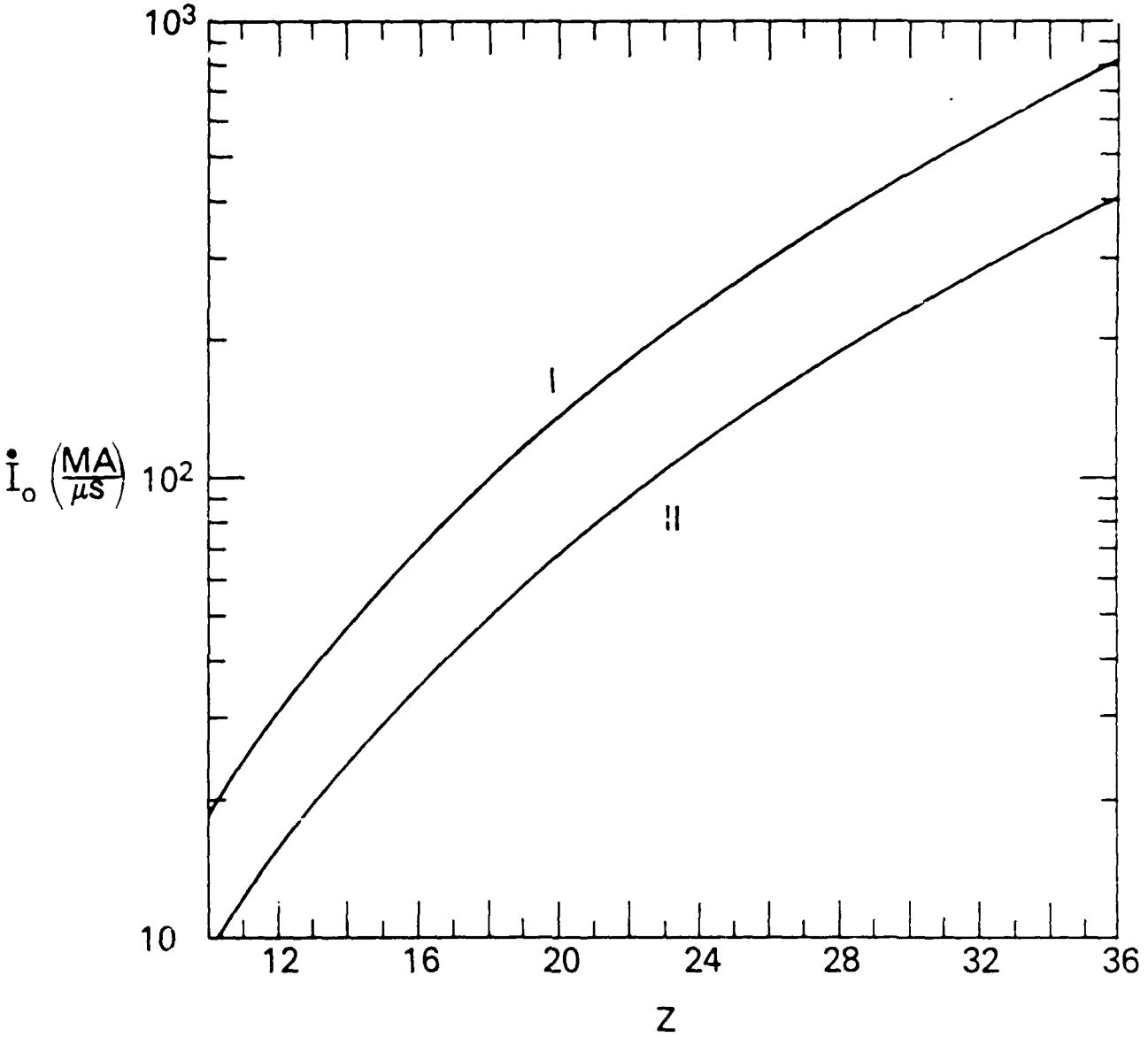

Figure 6. The scaling of the I-dot of the array implosion as a function of atomic number is shown for the two cases (I and II) described in the text.

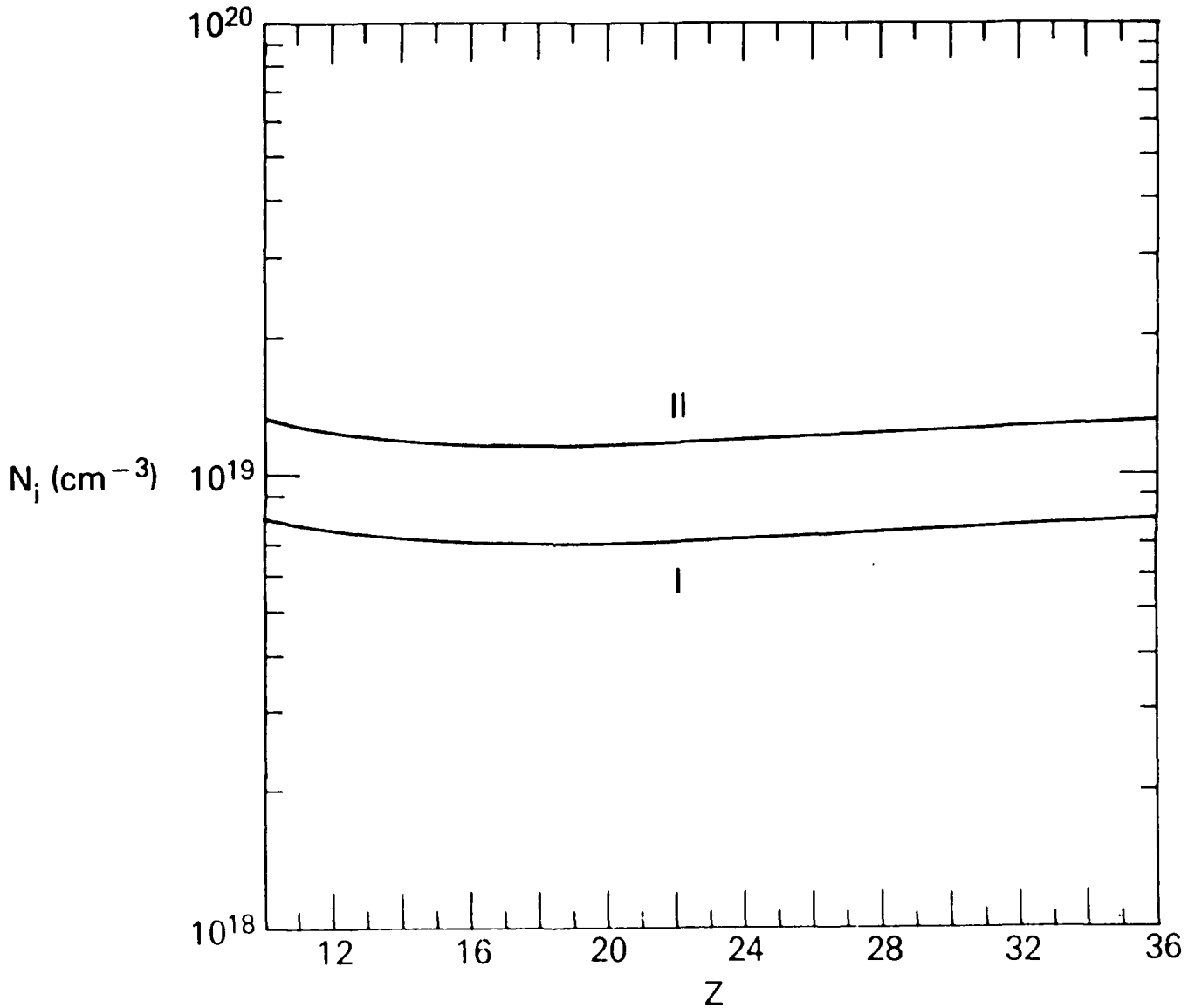

Figure 7. The final ion density that is achieved in the two array implosions, I and II, is shown as a function of atomic number.

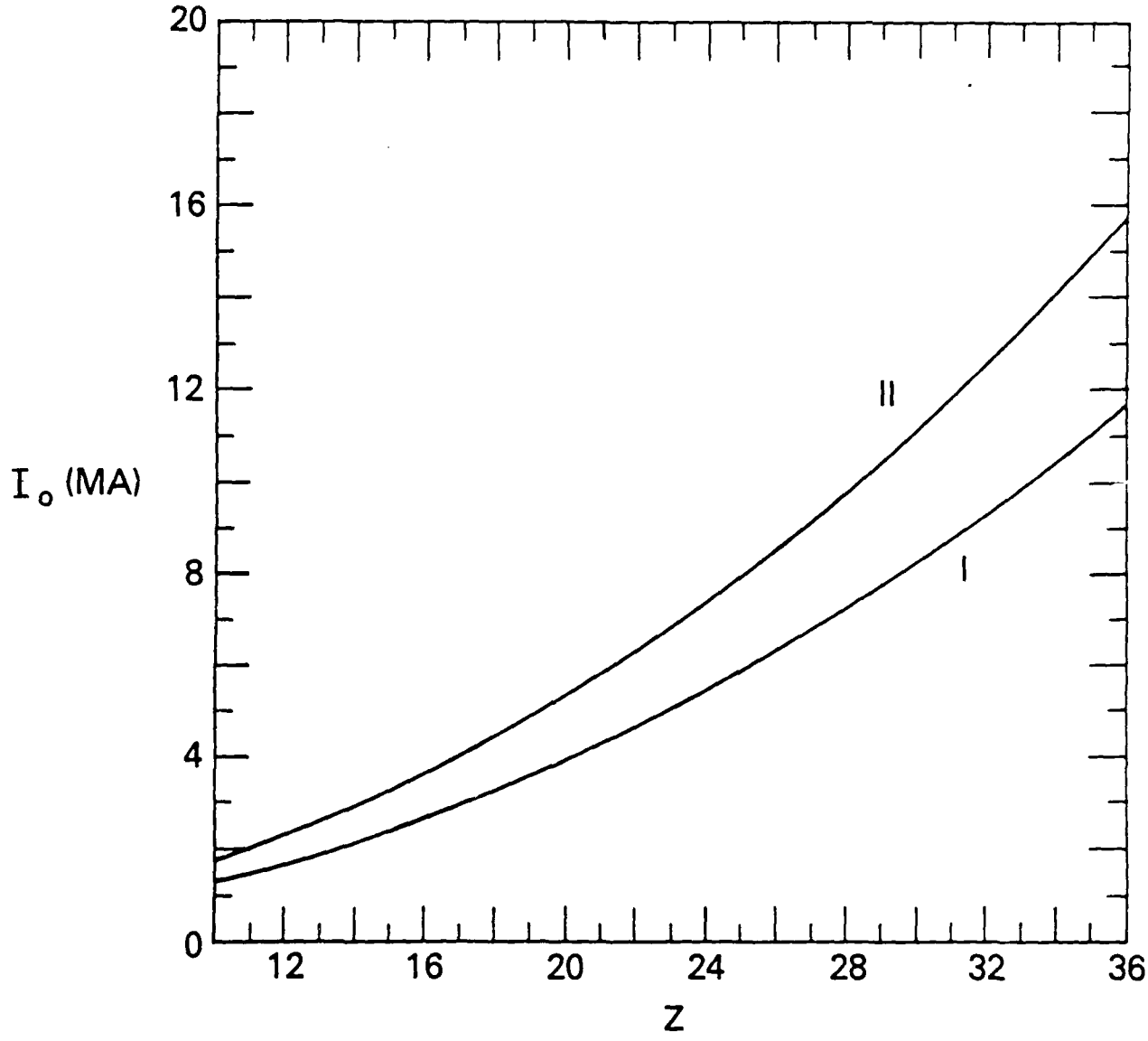

Figure 8. The peak current that is needed at the end of the array implosion in the two cases, I and II, to sustain a linear growth in current is shown as a function of atomic number.

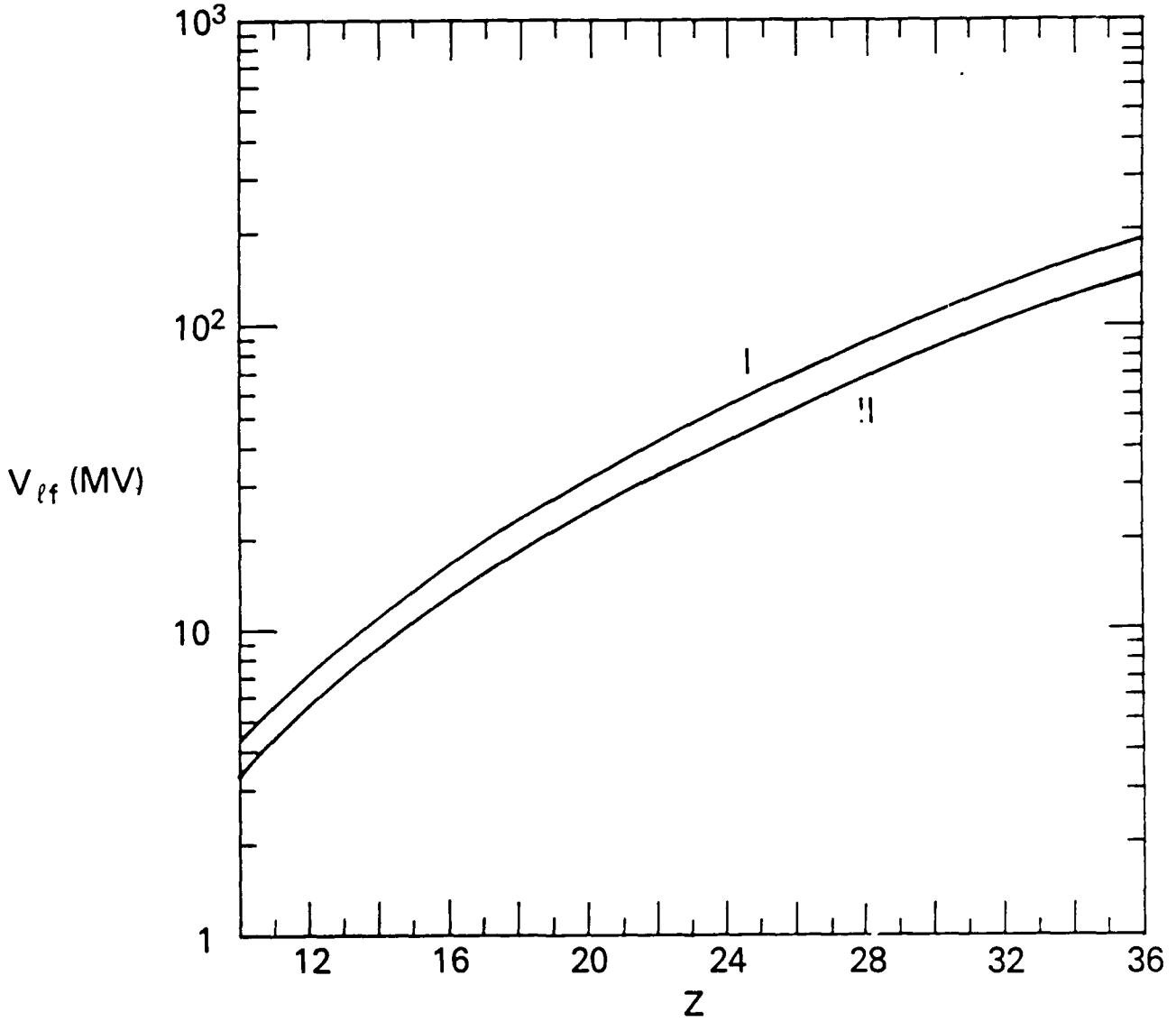

Figure 9. The final voltage that is impressed across the z-pinch in order to drive the peak current through the plasma in the two cases described in the text is shown as a function of atomic number.

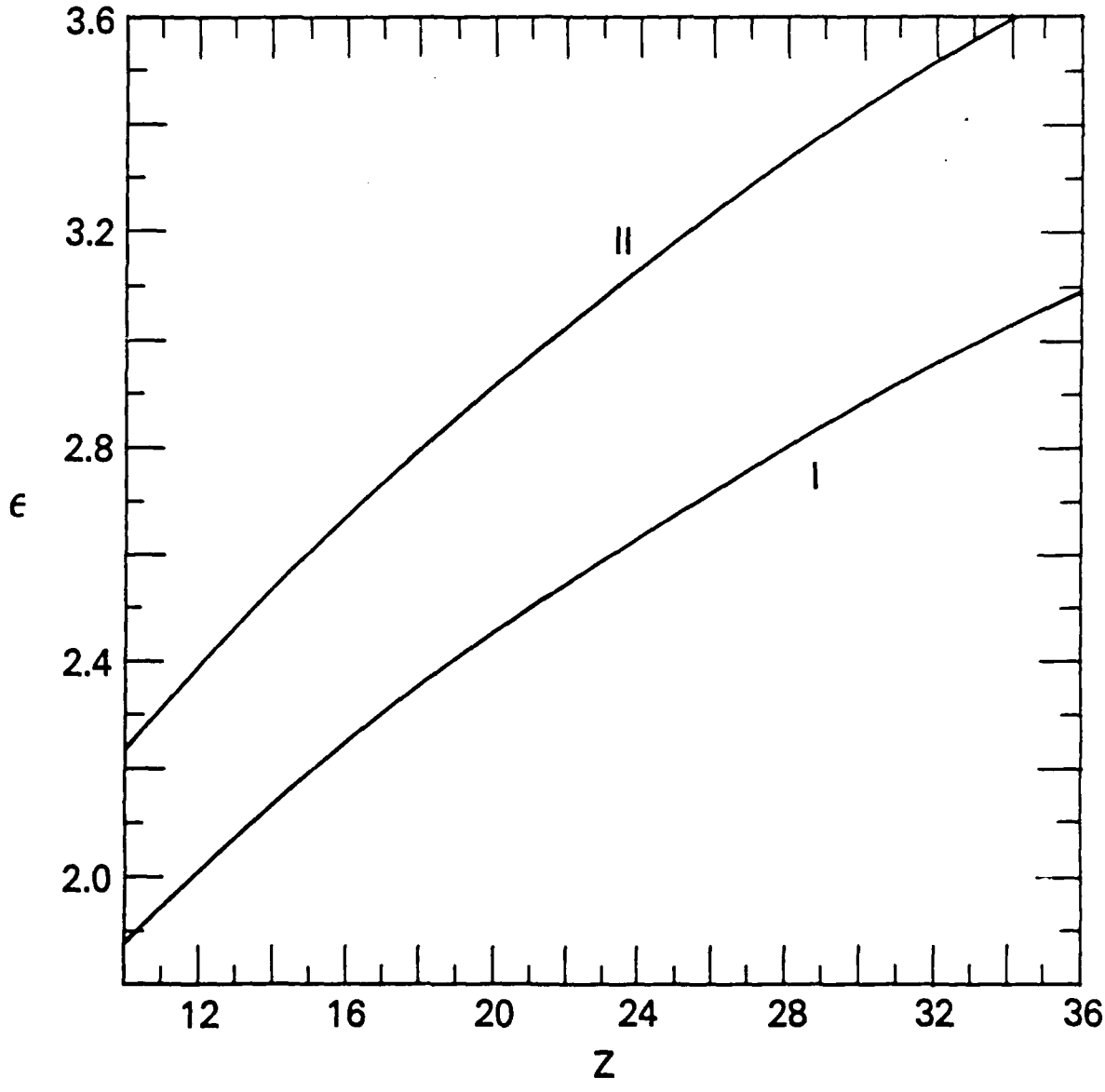

Figure 10. The thermalization efficiency that is needed to achieve force balance at the final radius of the implosion is shown as function of atomic number, for the two cases, I and II.

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